HEAT TRANSFER IN A WALL WHOSE HEAT INSULATION HAS BEEN COMPLETELY PENETRATED BY THE STRESS-BEARING ASSEMBLY

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We propose a method of calculating the steady-state heat transfer in structures in which the heat insulation has been cut through in its entirety by the stress-bearing assembly, with the latter an excellent conductor of heat. We present results from the experimental studies of heat transfer, and these differ from the theoretical data within limits of 10%.

Because of specific requirements dealing with strength, dimensions, etc., certain barrier constructions in isothermal enclosures are designed to include a heat-conducting stress-bearing assembly which cuts entirely through the heat insulation ("thermal bridges").

The method for the calculation of heat transfer in the case of complex walls in which the stress-bearing assembly does not cut all the way through the heat insulation have been studied in detail in [1-3]. Here the thermal resistance of those segments with the stress-bearing assembly and that of the heat insulation are of the same order of magnitude, so that the problem can be reduced to one that is one-dimensional. In the case of complete cutting through of the heat insulation by the thermal bridges, the thermal resistances of the corresponding segments differ sharply, and the problem is no longer one-dimensional. The use of the zone or layer method yields a divergence in the heat-transfer coefficient and in the calculation of heat leakage that may be as great as an order of magnitude. Other methods (circular flows, the Zhilinskii method, etc.) are entirely unsuitable. It is shown in [4] that the transfer of heat in such structures can be calculated only after the experimental determination of the temperature field within that structure, i.e., not during the layout stage, by which time the wall has already been fabricated.

The transfer of heat in complex structures with heat insulation can be calculated by methods making use of the analogy between electricity and heat, or by resort to digital computers [5, 6].

The proposed calculation method is based on the assumption that the entire flow of heat passes from one medium to another through the structural elements of the wall that are so highly conductive in terms of heat. This is valid when there exists a substantial difference between the thermal conductivity of the insulation and that of the metal. The inner and outer shells thus represent something on the order of extended surfaces with respect to the stress-bearing assembly. The diagram of the heat-transfer process and the temperature field in one of the repeating wall elements are shown in Fig. 1.

The following assumptions are made.

1. The heat is supplied uniformly to the inside metal shell, and it is removed with equal uniformity from the outside shell, i.e., the heat-transfer coefficients α_{I} and α_{II} are constant along the length of the shells.

2. There is no transfer of heat through the insulation. The flow of heat between the shells comes about exclusively through the metal elements of the stress-bearing assembly.

3. There is no transfer of heat at the side boundaries of the contour (lines 1-4 and 2-3).

4. There is no temperature difference across the thickness of the wall.

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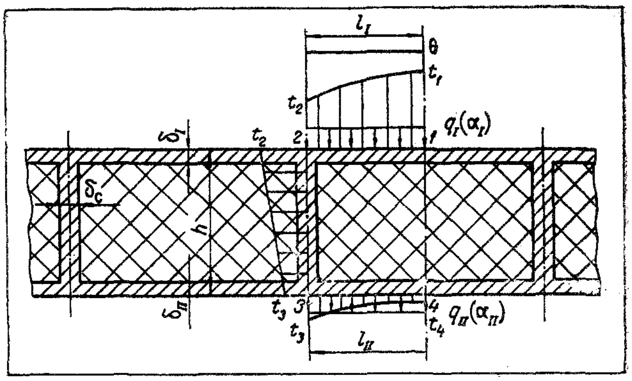

Fig. 1. Theoretical diagram showing the transfer of heat in a wall that has been entirely cut through.

We will write the heat-conduction equation for the segment l_I (the dimension in the direction perpendicular to the plane of Fig. 1 is assumed equal to 1 m):

$$\frac{d^2\vartheta}{dx^2} = \frac{\alpha_I}{\lambda_I \delta_I}\vartheta = m_I^2\vartheta. \tag{1}$$

Point 1 is taken as the coordinate origin, and it is situated on the shell, midway between the elements of the stress-bearing assembly ($x_1 = 0$, $x_2 = l_I$).

The solution of (1) has the form

$$\vartheta = \frac{\operatorname{sh} m_I (l_I - x)}{\operatorname{sh} m_I l_I}\vartheta_1 + \frac{\operatorname{sh} m_I x}{\operatorname{sh} m_I l_I}\vartheta_2$$

or

$$t = \frac{\operatorname{sh} m_I l_I - \operatorname{sh} m_I (l_I - x) - \operatorname{sh} m_I x}{\operatorname{sh} m_I l_I}\theta + \frac{\operatorname{sh} m_I (l_I - x)}{\operatorname{sh} m_I l_I} t_1 + \frac{\operatorname{sh} m_I x}{\operatorname{sh} m_I l_I} t_2. \tag{2}$$

The heat flow from the medium to the surface l_I is given by

$$Q = \alpha_I l_I (\theta - \bar{t}_I) = \alpha_I l_I \frac{\operatorname{ch} m_I l_I - 1}{m_I l_I \operatorname{sh} m_I l_I}(2\theta - t_1 - t_2). \tag{3}$$

The heat flow at point 2 ($x = l_I$) is given by

$$Q = -\lambda_I \delta_I \left(\frac{dt}{dx}\right)_{x=l_I} = -\lambda_I \delta_I \frac{m_I}{\operatorname{sh} m_I l_I}\left[\theta - t_1 - (\theta - t_2)\operatorname{ch} m_I l_I\right]. \tag{4}$$

In the vertical fin the temperature varies linearly, and

$$Q = \frac{\lambda_s \delta_s}{h}(t_2 - t_3). \tag{5}$$

For the segment l_{II} we have, in analogy with segment l_I, the relationships

$$t = \frac{\operatorname{sh} m_{II} x}{\operatorname{sh} m_{II} l_{II}} t_4 + \frac{\operatorname{sh} m_{II} (l_{II} - x)}{\operatorname{sh} m_{II} l_{II}} t_3; \tag{6}$$

here x is reckoned from the beginning of segment l_{II}, i.e., from point 3 ($x_3 = 0$, $x_4 = l_{II}$), and the heat flow at point 3 ($x = 0$) is given by

$$Q = -\frac{\lambda_{II}\delta_{II} m_{II}}{\operatorname{sh} m_{II} l_{II}}(t_4 - \operatorname{ch} m_{II} l_{II} t_3); \tag{7}$$

the heat flow from the surface l_{II} is given by

$$Q = \alpha_{II} l_{II} \bar{t}_{II} = \alpha_{II} l_{II} \frac{\operatorname{ch} m_{II} l_{II} - 1}{m_{II} l_{II} \operatorname{sh} m_{II} l_{II}}(t_3 + t_4). \tag{8}$$

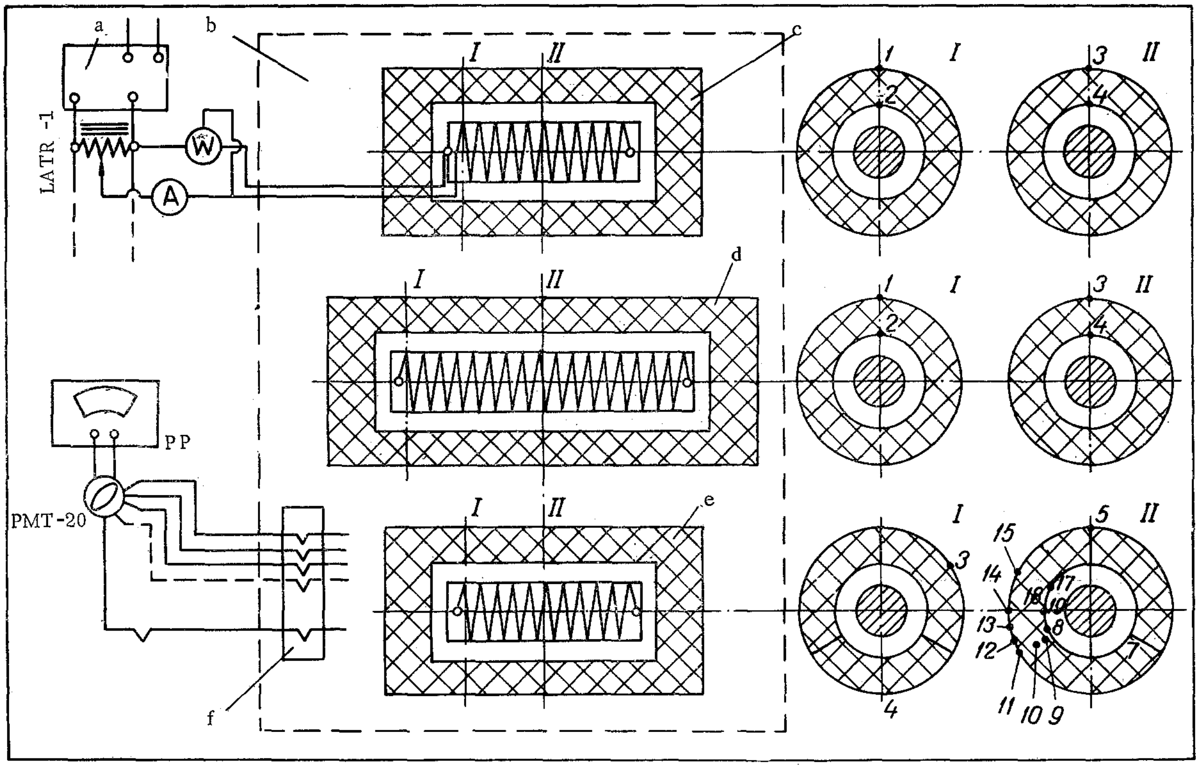

Fig. 2. Measurement scheme: a) stabilizer; b) heat chamber; c) short model; d) long model; e) model with fins; f) cold-junction blocks.

Simultaneous solution of (3), (4), (7), and (8), given equality of the heat flows, yields

$$t_1 = \theta - \frac{m_I Q}{\alpha_I \operatorname{sh} m_I l_I}, \quad (9)$$

$$t_2 = \theta - \frac{m_I Q}{\alpha_I \operatorname{th} m_I l_I}, \quad (10)$$

$$t_3 = \frac{m_{II} Q}{\alpha_{II} \operatorname{th} m_{II} l_{II}}, \quad (11)$$

$$t_4 = \frac{m_{II} Q}{\alpha_{II} \operatorname{sh} m_{II} l_{II}}. \quad (12)$$

The heat flow passing through the barrier element under consideration is

$$Q = \theta \Big/ \left(\frac{h}{\lambda_s \delta_s} + \frac{m_I}{\alpha_I \operatorname{th} m_I l_I} + \frac{m_{II}}{\alpha_{II} \operatorname{th} m_{II} l_{II}} \right). \quad (13)$$

Thus, with the above-cited formulas we can calculate the temperature distribution through the heat-conducting stress-bearing assembly cutting through the heat insulation, and we can also calculate the quantity of heat passing through this "thermal bridge." It should be noted that these formulas are suitable both for flat walls (when $l_I = l_{II} = l$) as well as for cylindrical walls and walls that are not completely joined (when $l_I \neq l_{II}$). The thickness δ can be varied along the length l. The heat-transfer coefficient for the flat wall is equal to

$$k = \frac{Q}{l\theta}. \quad (14)$$

As an example, in Table 1 we compare the heat-transfer coefficients calculated by various methods for a flat wall, with the following initial data: $l_I = l_{II} = 0.146$ m, $\delta_I = \delta_{II} = \delta_s = 2 \cdot 10^{-3}$ m, h = $76 \cdot 10^{-3}$ m, $\alpha_I = 5.82$ W/m^2·deg; $\alpha_{II} = 23.3$ W/m^2·deg, $\lambda_m = 46.6$ W/m·deg, $\lambda_{in} = 0.0466$ W/m·deg.

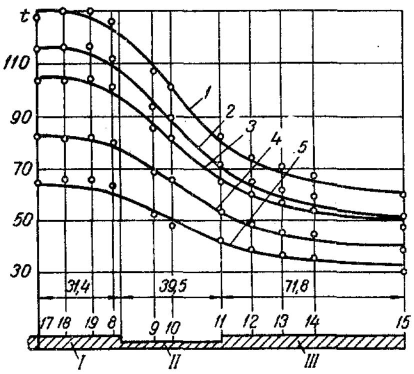

Fig. 3. Distribution of temperature t, °C, in those elements of the model with fins (solid lines denote theory; points denote experimental data): Key: 1) W = 325 W; 2) 290 W; 3) 250 W; 4) 200 W; 5) 150 W; I) inside skin; II) fin; III) outside skin; the numerals along the horizontal lines denote the numbers of the thermocouples.

TABLE 1. Comparison of Theoretical Heat-Transfer Coefficients, $W/m^2 \cdot deg$

Method	Heat-transfer coefficient	Method	Heat-transfer coefficient
Zone	0,593	Proposed	2,18
Layer	3,14	Numerical [7]	2,13

We see from Table 1 that the results of the calculation based on the proposed approximate method differ from the calculation results with the "exact" numerical method by no more than 2%.

The above-derived relationships were tested on models in a heat chamber.

The models are made of two concentric brass shells (L62 brass, 1.5 mm in thickness, with a coefficient of thermal conductivity $\lambda_m = 116.3$ W/m·deg), with the cavity between these shells filled with asbestos insulation. The ends are insulated with asbestos that is 40 mm thick. Nichrome electric heaters with a resistance of 110 Ω are placed inside the models; these heaters are wound onto fiber glass-insulated steel tubes with a diameter of 20 × 2. We investigated three models:

1) a model 500 mm in length, with 3 fully penetrating brass fins 1.5 mm in thickness, connecting the model shells and cutting through the asbestos insulation (the model with fins);

2) a model 500 mm in length without fins (the short model);

3) a model 800 mm in length without fins (the long model).

The last two models are needed to provide for the effect of the ends, for the determination of heat-flux density at the cylindrical portion of the model, and for the determination of the coefficient of thermal conductivity for the asbestos. In the tests, all three models were placed into the heat chamber simultaneously.

The heaters were powered from an ac circuit of 220 V through a voltage stabilizer and the current was regulated by means of an LATR-1 autotransformer. The heater power was measured with a 0.5 glass wattmeter. A temperature from −10° through −50°C was maintained within the chamber, with an accuracy of ±1°C.

The temperature was measured at various points on the models by means of Chromel–Copel thermocouples 0.5 mm in diameter, connected through multiple PMT-20 switches to portable PP potentiometers (Fig. 2). The hot thermocouple junctions were in a mass of asbestos or embedded in special depressions in the brass fins and in the shells of the models, and then they were sealed. The cold junctions were held in special terminal blocks in the heat chamber, with the copper wiring to the potentiometer from the cold thermocouple junctions leading out of the chamber through the packing of the outside wall. The sought temperature was thus reckoned with respect to the temperature of the chamber, which was constantly being monitored by means of the thermocouple, one of whose junctions was in the area of the terminal block with the cold junctions, while the other junction was within the enclosure, and the temperature of the chamber was also measured by means of the thermocouple which was connected to the control panel for the chamber.

On the basis of the results derived here, we can determine the nature of heat transfer in an insulated structure kept by a metallic crosspiece in the case of a steady-state regime.

Table 2 shows the temperature distribution for various points on the models, reckoned from the temperature of the chamber for various heat-release rates and temperatures within the chamber (see Fig. 2).

The coefficient of thermal conductivity for the asbestos is determined from comparison of measurements conducted on long and short models on the assumption that the heat losses through the ends are equal for an identical temperature difference across the cylindrical segments and that it is equal to $\lambda_{in} = 0.35$ W/m·deg, which is somewhat higher than the tabulated values, and this is evidently a consequence of elevated moisture content.

TABLE 2. Experimental Results

Experiment number		I	II	III	IV	V
chamber temperature °C		−48,8	−50,0	−11,6	−40,0	−39,4
	Points	Long model				
Power, W		60	300	200	250	170
Temperature, °C	3	24,8	47,7	29,6	34,8	22,0
	4	27,4	143,0	94,3	113,3	75,8
		Short model				
Power, W		40	250	200	170	210
Temperature, °C	3	7,8	54,6	41,8	36,2	47,0
	4	24,8	164,3	131,7	111,8	138,4
		Model with fins				
Power, W		250	150	200	325	290
Temperature, °C	8	101,4	63,8	79,7	126,4	112,2
	9	85,2	54,0	68,2	106,6	94,6
	10	81,4	48,6	66,0	101,6	90,2
	11	65,2	42,1	53,0	82,4	72,4
	12	60,0	38,5	48,2	74,8	65,2
	13	57,6	37,3	45,8	71,6	62,2
	14	54,8	35,5	44,8	67,6	58,8
	15	48,4	31,8	39,5	61,2	52,6
	16	69,3	43,8	55,6	86,6	75,8
	17	104,2	65,4	82,0	127,4	116,0
	18	104,2	65,9	82,2	130,2	115,8
	19	104,4	65,6	82,6	130,6	116,8

In the experiment, heat transfer in the model occurs under boundary conditions of the second kind on the segment l_I and under boundary conditions of the third kind at the l_{II} surface. The temperature field in this case is determined from the following formulas:

for the inside surface

$$t = \frac{ql_I^2}{2\lambda_I \delta_I}\left(1 - \frac{x^2}{l_I^2}\right) + \frac{ql_I m}{\alpha \operatorname{th} ml_{II}}, \tag{15}$$

for the outside surface

$$t = \frac{ql_I m}{\alpha \operatorname{sh}^2 ml_{II}}\left[\operatorname{sh} mx + \operatorname{sh} m(l_{II} - x)\operatorname{ch} ml_{II}\right]. \tag{16}$$

The experimental results were processed in the following manner. A curve was plotted to show the power of the heaters as a function of the maximum temperature difference Δt_{max} for all models, i.e., W = f(Δt_{max}), from which, for each experimental value of the power W_f for models with fins, we determine the magnitude of the power (W_1) for the long model and the power (W_s) for the short model, given the same temperature difference Δt_{max}. Then a comparison of the long and short models makes it possible to determine the fraction of the power for the cylindrical portion and for the end leakages for the short model, which are assumed to be equal to the end leakages of the model with fins, given the same value for Δt_{max}. We then determine the fraction of heat through the cylindrical portion of the model with fins (as the difference between the total power and the end losses) and the heat-flux density q referred to the inside surface. From the experimentally known temperature field and heat flow at the outside cylindrical surface of the model with fins we determine the heat-transfer coefficient α.

Figure 3 shows the results from the calculation of the temperature distribution along the stress-bearing assembly of the models at various powers.

As we can see from the curve, the difference between the theoretical and experimental data on temperature does not exceed 10%. Consequently, the assumptions contained in this calculation scheme are more valid than in the zone and layer methods for the case of complete penetration of the heat insulation, and the calculation results are more accurate.

NOTATION

x is the coordinate along the metal surface, m;
l is the theoretical length of the skin, m;

h is the thickness of the wall or the height of the fin, m;
δ is the thickness of the metallic structure, m;
t is the temperature, °C;
θ is the temperature difference between the media surrounding the wall (the temperature in the enclosure, given a zeroth outside temperature), °C;
$\vartheta = \theta - t$ is the temperature difference between the medium and the surface, °C;
t_i ($i = 1, 2, 3, 4$) is the temperature at characteristic points in the structure, °C;
λ is the coefficient of thermal conductivity, W/m·deg;
α is the heat-transfer coefficient, W/m^2·deg;
$m = \sqrt{\alpha/\lambda\delta}$, m^{-1};
Q is the heat flow, W;
q is the heat-flux density, W/m^2;
k is the heat-transfer coefficient, W/m^2·deg;
W is the power, W.

Subscripts

I denotes the inside surface;
II denotes the outside surface;
s denotes the stress-bearing assembly;
m denotes the metal;
in denotes the insulation;
1, 2, 3, 4 denotes the characteristic points on the structure (see Fig. 1);
max denotes maximum;
f denotes the model fins;
l denotes the long model;
s denotes the short model.

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